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Monitoring of delamination defects

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The purpose of this paper is to propose an alternative approach to the inverse dynamic analysis problem. Generalizing the so-called VDM (Virtual Distortion Method) approach for dynamic problems, a dynamic influence matrix D concept will be introduced. Pre-computing of the time-dependent matrix D allows for decomposition of the dynamic structural response into components caused by external excitation in undamaged structure (the linear part) and components describing perturbations caused by the internal defects (the non-linear part). As a consequence, analytical formulae for calculation of these perturbations and the corresponding gradients can be derived. The physical meaning of the so-called *virtual distortions* used in this paper can be explained with the help of externally induced strains (non-compatible in general, e.g. caused by piezoelectric transducers, similarly to the effect of non-homogeneous heating). The compatible strains and self-equilibrated stresses are structural responses to these distortions. Assuming possible locations of all potential defects in advance, an optimisation technique with analytically calculated gradients could be applied to solve the problem of the most probable location of defects. The considered damage can affect the local stiffness as well as the mass distribution modification. It is possible to identify the position as well as intensity of several, simultaneously generated defects (delaminations).

1. Introduction

The damage detection systems based on array of piezoelectric transducers sending and receiving strain waves are intensively discussed by researchers recently. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem.

An alternative way for these techniques is the VDM approach. The software tool based on this method can be dedicated for different kind of damage, also for delamination defect identification problem, which will be discussed below.

2. VDM static approach in delamination monitoring

We shall pose the optimisation problem of structural damage identification (constraining ourselves temporarily to the static case) within the framework of the Virtual Distortion Method (cf. [4]). Let us minimise the following function:

$$\min \sum_A \left(\varepsilon_A^{\mathrm{M}} - \varepsilon_A\right)^2, \tag{2.1}$$

which can be interpreted as an average departure of the total structural strain ε_A from the in-situ measured strain $\varepsilon_A^{\mathrm{M}}$ in damaged locations A. Taking advantage of the VDM formulation we can decompose the strain ε_A into two parts:

$$\varepsilon_A = \varepsilon_A^{\mathrm{L}} + \varepsilon_A^{\mathrm{R}} = \varepsilon_A^{\mathrm{L}} + \sum_i D_{Ai}\, \varepsilon_i^0, \tag{2.2}$$

where $\varepsilon_A^{\mathrm{L}}$ denotes the response of undamaged structure, $\mathbf{D}$ is the influence matrix and ε^0 is the virtual distortion vector. As the component $\varepsilon_A^{\mathrm{L}}$ is constant for a given external load, the so-called residual strain component $\varepsilon_A^{\mathrm{R}}$ may only be varying in the optimisation process with the virtual distortion ε^0 as the design variable.

We shall measure the structural damage in each member i with the help of the coefficient μ_i i.e. with the ratio of cross-sectional areas of a damaged member to the undamaged one. Consequently we have to impose appropriate constraints on this coefficient. As we examine the physical process of deterioration of the member cross-section we are interested in such μ_i, which complies with the following constraints:

$$0 \leqslant \mu_i \leqslant 1, \quad \text{i.e.} \quad 0 \leqslant \frac{\varepsilon_i - \varepsilon_i^0}{\varepsilon_i} \leqslant 1. \tag{2.3}$$

For delamination problems the coefficient μ_i will finally (after optimisation) take only two values: 0 (delamination) or 1 (full connection).

The gradients of the objective function and the constraints are expressed in terms of the design variable ε^0 as follows:

$$\begin{aligned} \nabla f &= \frac{f}{\partial \varepsilon_k^0} = -2 \sum_A D_{Ak}\left(\varepsilon_A^{\mathrm{M}} - \varepsilon_A\right) \\ \text{and} \quad n_{kl} &= \frac{g_l}{\partial \varepsilon_k^0} = \frac{\delta_{lk}\, \varepsilon_l - D_{lk}\, \varepsilon_l^0}{\varepsilon_l^2}. \end{aligned} \tag{2.4}$$

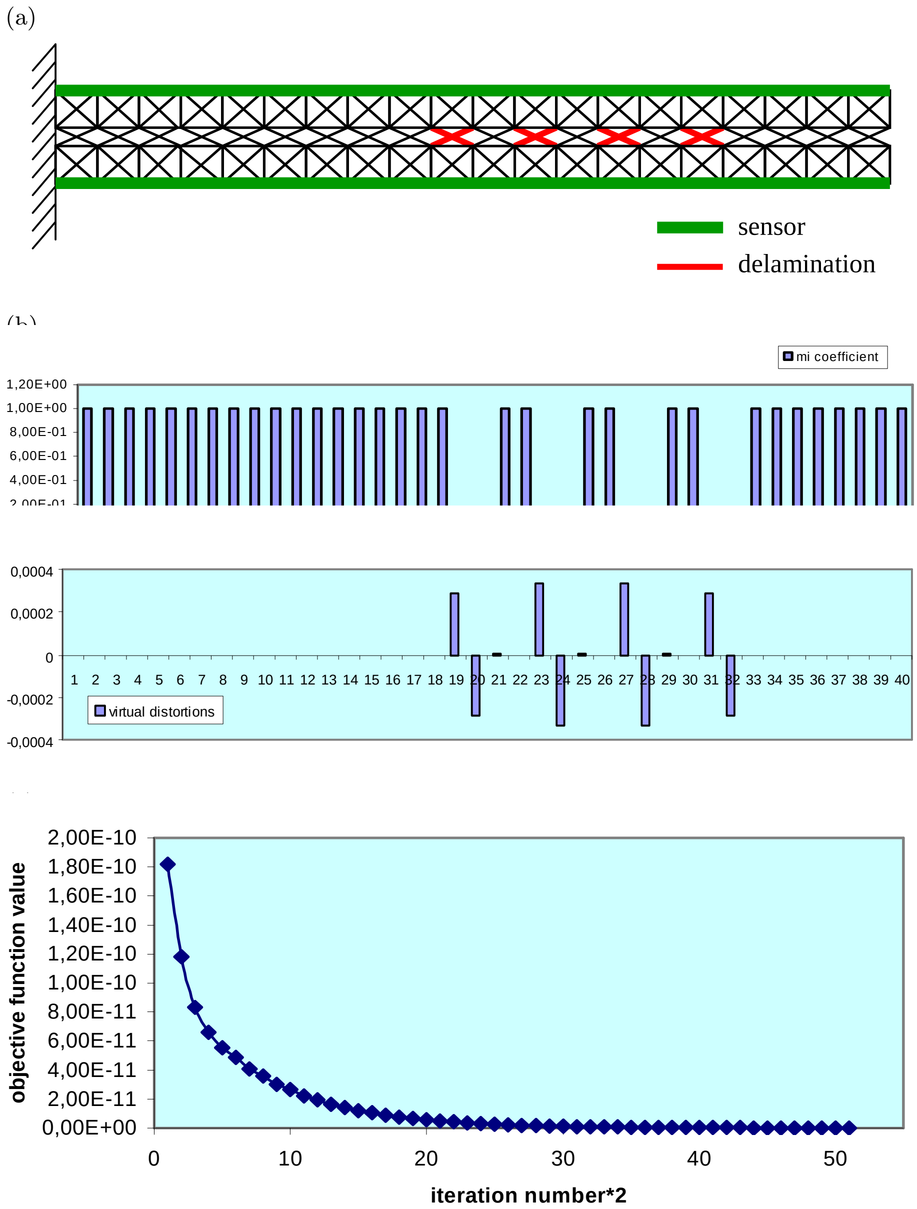

FIGURE 1. (a) Cantilever truss structure consisting of 2 outer layers joined by the inner layer, which exhibits delamination in few locations (red dashed lines). Sensors (elements able to detect strain) are placed in upper and lower horizontal members marked by bold green lines. Static vertical force was applied at free end to identify the delamination defects. (b) Virtual distortions and damage coefficient values obtained during optimization process. (c) Optimization routine progress.

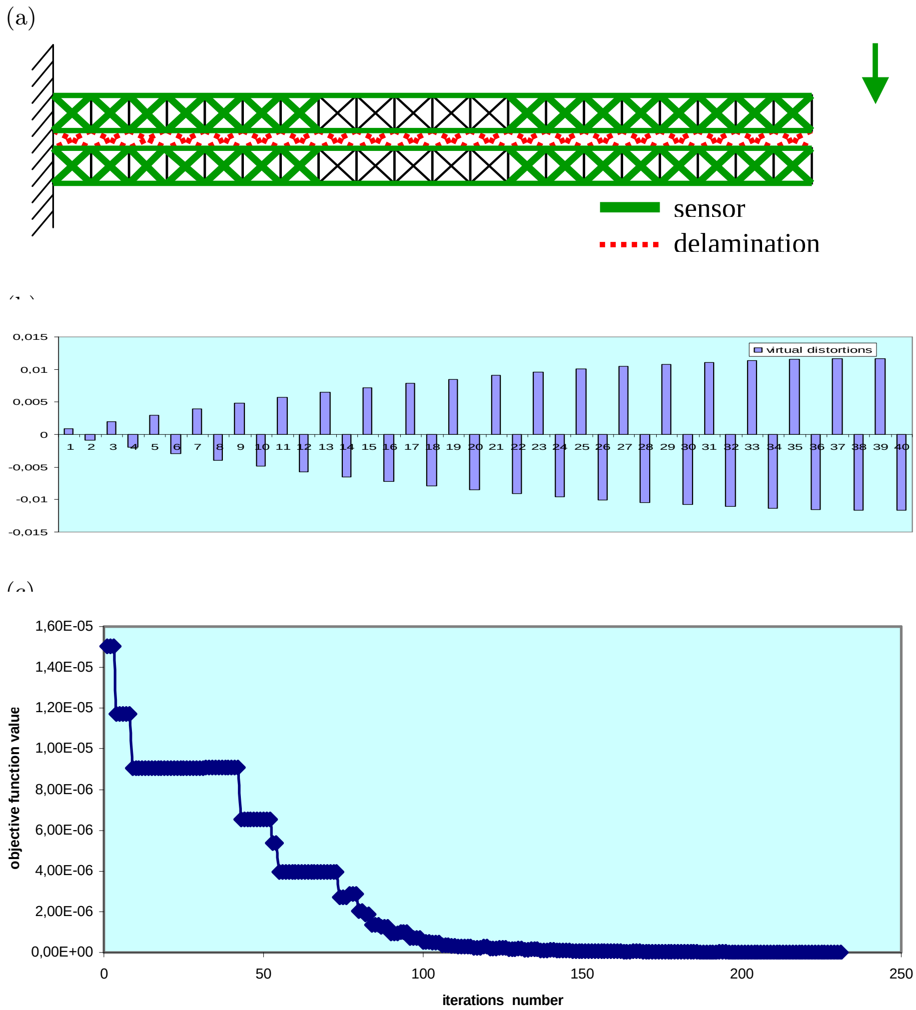

Figure 2. (a) Full delamination case. Sensors are marked by bold green lines. Static vertical force was applied at free end to identify the delamination defects. (b) Damage coefficient values obtained in optimization process. (c) Optimization routine progress.

In order to solve the damage identification problem posed by (2.1) and (2.3) the Gradient Projection Method (cf. [1, 2, 3]) can be used as optimisation tool. The Gradient Projection Method is based on the idea of projecting the search direction (i.e. the direction in which the objective function value decreases) into the subspace tangent to the active constraints. For the case of linear constraints the optimisation problem can be posed in the following way:

$$\min f(x), \tag{2.5}$$

subject to:

$$g_j(\mathbf{x}) = \sum_{i=1}^{n} n_{ji}\, x_i - b_j \geqslant 0, \quad j = 1, \ldots, n_g, \tag{2.6}$$

where $n_{kl} = \frac{g_l}{\partial x_k}$ i.e. the gradients of the constraints are stored column-wise. Subscripts i and k run through the number of design variables n whereas subscripts j and l run through the number of constraints n_g.

If we select only the r active constraints then the constraints may be written as follows:

$$\mathbf{g}^{\text{active}}(\mathbf{x}) = \mathbf{n}^{\mathrm{T}}\mathbf{x} - \mathbf{b} = \mathbf{0}, \tag{2.7}$$

where the matrix $\mathbf{n}$ stores gradients of the constraints in columns.

3. Numerical example

A simple truss model has been used demonstrating the problem of identification of delamination zone. The delaminated region has been modelled as a very thin layer (composed of truss elements also) placed between two thick layers.

Numerical tests have been done for different delamination zone positions and sizes. Optimization routine was successful in finding defects, but large number of sensors was used, especially in the case of full delamination effect.

4. Dynamic case

The problem of identification of delaminations defined as a static problem leads to multi-sensor *observability*, which was demonstrated above. Let us demonstrate now, that the same problem, defined as a dynamic one, allows us to use only few sensors. Assuming impact excitation (generated by an actuator), transmitted along the beam and measured by a sensor located in a distance, the inverse dynamic analysis has to be performed in order to identify locations and intensities of defects. The Dynamic Virtual Distortion Method [4], which is based on assumption that the virtual distortions depend on time, can be applied as the solver of the current identification problem. Both the structural response and influence matrix are time-dependent, and the formula for measured strain development (2.2) takes now the following form:

$$\varepsilon_A(t) = \varepsilon_A^{\mathrm{L}}(t) + \varepsilon_A^{\mathrm{R}}(t) = \varepsilon_A^{\mathrm{L}}(t) + \sum_{\tau=0}^{t} \sum_{i} D_{Ai}(t-\tau)\, \varepsilon_i^0(\tau). \tag{4.1}$$

It is important to note, that time-dependent influence matrix is obtained for unit impulse excitations applied in time instant t=0 (it means that excitation has non-zero value only for one time step). The unit impulse excitation can be supplied in form of initial velocity conditions: $V(0) = \frac{P\Delta t}{m}$, where P denotes compensative force corresponding to locally generated unit virtual distortion impulse $\varepsilon^0 = 1$, Δt denotes the integration time step, and m is the mass concentrated in the loaded element. Having the influence matrix $D_{Ai}(t)$ (in the case of only one sensor $D_{Ai}(t)$ describes strain in observable element A, for each possible location of compensative forces) we can calculate the superposition of linear, time-dependent structural responses.

The inverse analysis leads to minimisation of the objective function describing differences between the measured response $\varepsilon_A^{\mathrm{M}}$ and the modelled one ε_A (expressed by Eq. 4.1). Finally, the defect identification problem takes the following form:

$$\min \sum_t \left(\varepsilon_A^{\mathrm{M}}(t) - \varepsilon_A(t)\right)^2, \tag{4.2}$$

subject to the following constraints, where μ_i is defined by the time-dependent version of the formula (2.3):

$$0 \leqslant \mu_i \leqslant 1. \tag{4.3}$$

5. Conclusions

Potential of use of the VDM-based approach to identification of delamination defect has been presented. It has been demonstrated that a relatively large number of sensors has to be applied in some cases in order to identify properly delamination defects in case of statically formulated problem. Let us now demonstrate (using the same, relatively simple numerical model) that the dynamic problem formulation is promising thanks to placing only few sensors for damage identification.

A smaller, 98-element truss structure model has been used in the dynamic case presented below. Full sine impulse load has been applied in the tip point of the truss-beam cantilever structure (Fig. 3(a)) to generate the reference response as well as the response of damaged structure. Then, the steepest descent method with analytically computed, VDM-based gradients has been used to perform the minimization process (Fig. 3(b, c)). The dynamic approach proposed above will be developed and tested numerically on the basis of a more precise structural model in a separate paper.

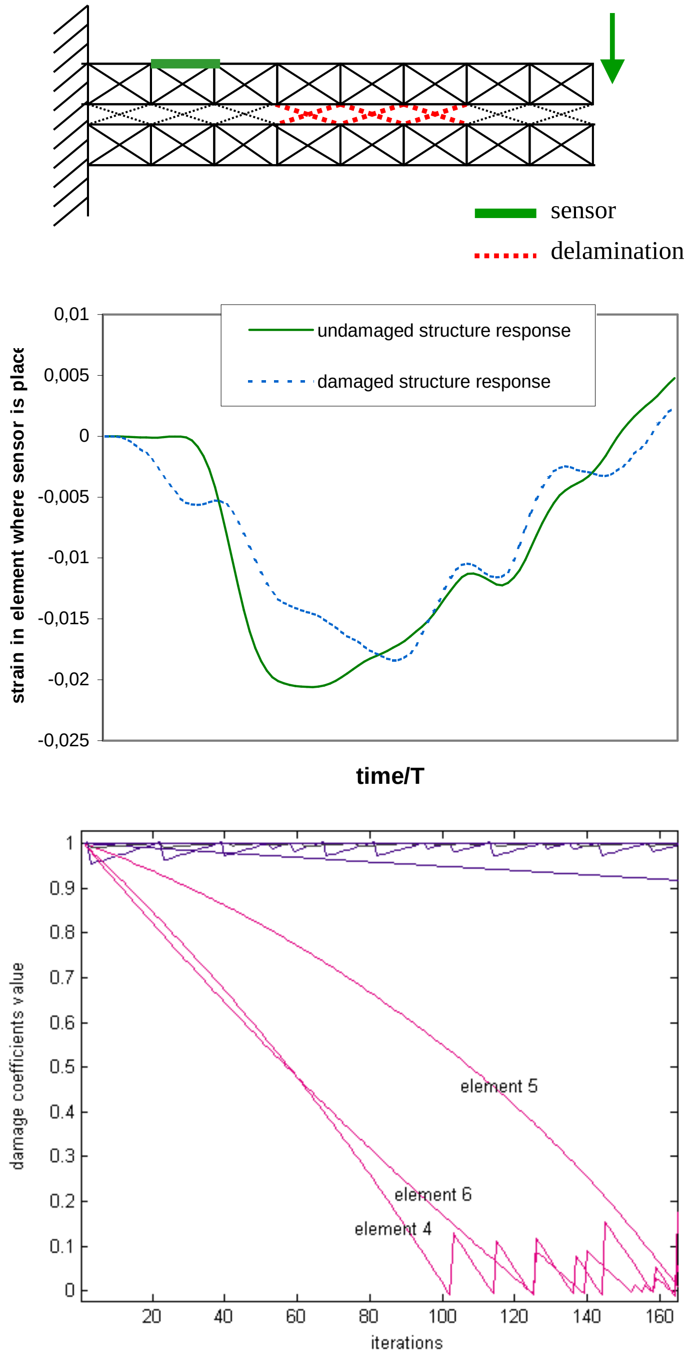

FIGURE 3. Dynamic VDM approach results: (a) the numerical model; (b) damaged and undamaged structure responses; (c) inverse analysis effect.

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